

High Scale Physics Connection to LHC Data¹

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Abstract

The existing data appears to provide hints of an underlying high scale theory. These arise from the gauge coupling unification, from the smallness of the neutrino masses, and via a non-vanishing muon anomaly. An overview of high scale models is given with a view to possible tests at the Large Hadron Collider. Specifically we discuss here some generic approaches to deciphering their signatures. We also consider an out of the box possibility of a four generation model where the fourth generation is a mirror generation rather than a sequential generation. Such a scenario can lead to some remarkably distinct signatures at the LHC.

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Introduction: The standard model of electro-weak interactions is remarkably successful in explaining experimental data up to LEP energies, of ~ 100 GeV. However, this model cannot be extrapolated to the Planck scale,

$$M_{Pl} = (8\pi G_{Newton})^{-1/2} \sim 2.4 \times 10^{18} \text{ GeV}. \quad (1)$$

One of the purposes of high energy physics is to understand the laws of physics from low energy up to M_{Pl} . Therefore much of the current effort in high energy physics is focussed on discovering what lies beyond the standard model. Although the standard model is remarkably successful it does suffer from some drawbacks. One obvious drawback is the appearance of many arbitrary parameters. On a more theoretical level the loop correction to the Higgs mass diverges quadratically, and there is a lack of unification with gravity. Other things the standard model does not explain is why the charges of quarks and leptons are quantized, and why the matter in the universe is made up mostly of protons and not of anti-protons, and what gives rise to non-baryonic dark matter and dark energy. Further, one might ask what explains the mass hierarchy. A variety of possibilities have been considered as one goes beyond the standard model. These include grand unification (GUT), compositeness, supersymmetry (SUSY) and supergravity (SUGRA) based models, strings and branes, extra (warped) dimensions, AdS/CFT, Stueckelberg and other U(1) extensions, unparticles, noncommutative geometry and many other possibilities. These extensions may be loosely classified as falling in two broad approaches: the top down and the bottom up. In the top down approach one starts with a presumed candidate for a unified theory and one works one's way downward to realize the standard model. In the bottom up approach, one starts at the electroweak scale and works upwards to high scales which could be either the grand unification scale or the string/ Planck Scale.

High scale theories: There are at least two important hints that one is dealing with high scale models as one goes beyond the standard model. The first of these is the unification of gauge coupling constants. Here one finds that the extrapolation of high precision LEP data produces a unification of the $SU(3)_C \times SU(2)_L \times U(1)$ gauge couplings g_3, g_2, g_1 at a high scale within SUGRA soft breaking models with the MSSM spectrum. (For reviews of high scale physics see [1, 2]). While unification of gauge couplings is also possible within the extra dimensions models due to a power law running, it is less predictive in that context. Another hint of a high scale arises from the smallness of neutrino masses. The astrophysical limits

from the WMAP satellite experiment on the sum of neutrino masses is[3]

$$\sum_{i=1,2,3} |m_{\nu_i}| < .7 \text{ eV}. \quad (2)$$

A small neutrino mass O(eV) could arise from a See-Saw mechanism[4]

$$m_{\nu} \sim \frac{m^2}{M} \quad (3)$$

With $m \sim M_W \sim 10^2 \text{ GeV}$ and $M \sim 10^{16} \text{ GeV}$, one can generate neutrino masses in the sub eV region. Thus the smallness of neutrino masses points to a high scale within the SeeSaw mechanism. While small neutrino masses can also be generated by other mechanisms, the SeeSaw mechanism appears to be more natural one.

High scale physics points to SUSY: If one wishes to incorporate high scales along with the standard model in a common framework, then supersymmetry appears to be a logical possibility. The reason for this is because SUSY stabilizes scales. Incorporation of gravity in SUSY requires transition from global susy to local supersymmetry[5] and its modern form supergravity[6]. SUSY grand unification incorporates GUTs and SUSY but has no gravity. Supergravity grand unification has SUSY, GUTs and gravity. However, a viable scheme requires breaking of supersymmetry and in supergravity grand unification (SUGRA GUT)[7] supersymmetry is broken by gravity mediation[7, 8, 9]. An interesting point about supergravity is that it is the field point limit of string theory, and thus check on the validity of SUGRA GUT would be a pointer to the underlying unified theory of quantum gravity.

Implications of high scale models at the electroweak scale: In a supersymmetric model or string model, to make contact with low energy physics one must break the supersymmetry with soft terms, and typically the generation of soft terms involves three steps: (i) First one must generate SUSY breaking in a sector different from the visible sector; (ii) Then this breaking is communicated by messengers to the visible sector; and finally (iii) With (i) & (ii) one generates soft terms in the visible sector. The two main scenarios for supersymmetry breaking are the gravity mediated breaking[7, 8, 9, 10], and gauge mediated breaking[11]. The breaking of SUSY in the hidden sector could be arranged by a variety of methods. The simplest way it by use of a chiral superfield in the hidden sector. In this case the

one finds

$$m_{soft} \sim F/M_{\text{Pl}}, \quad (4)$$

where with $\sqrt{F} \sim 10^{10}$ GeV, one finds $m_{\text{soft}} \sim 10^{2-3}$ GeV. Further, $m_{\text{soft}} \sim 10^{2-3}$ GeV can also arise from gaugino condensation[12] such that $m_{\text{soft}}^{\text{string}} \sim \langle \lambda\lambda \rangle / M_{\text{Pl}}^2$, $\langle \lambda\lambda \rangle \sim (10^{13} \text{ GeV})^3$. In gauge mediated breaking one finds that soft SUSY breaking in the visible sector is generated radiatively[11]

$$m_{\text{soft}} \sim (\alpha/4\pi)F/M_{\text{msg}}, \quad (5)$$

where with $\sqrt{F} \sim M_{\text{msg}} \sim 10^4$ GeV, one has $m_{\text{soft}} \sim (10^2 - 10^3)$ GeV. A variety of other possibilities also exist for the breaking of supersymmetry such as anomaly breaking, hierarchical breaking of SUSY, and meta -stable supersymmetry breaking. We will discuss the hierarchical breaking of SUSY a bit later. Sugra and heterotic string models are high scale models. The model mSUGRA is defined by the soft terms $V_{SB} = m_0^2 \sum_i \phi_i^\dagger \phi_i + A_0 W^{(3)} + B_0 W^{(2)} + m_{1/2} \sum_{\alpha=3,2,1} \bar{\lambda}_\alpha \lambda_\alpha$, where $W^{(2)}$ is the quadratic part and $W^{(3)}$ is the cubic part of the superpotential. In MSSM $W^{(2)}$ takes the form $W^{(2)} = \mu \epsilon_{ij} H_1^i H_2^j$, where H_2 couples to the up quarks and H_1 couples to the down quarks and the leptons. Thus the parameter space of mSUGRA model is defined by $m_0, m_{1/2}, A_0, B_0$, and μ . An interesting phenomenon in SUGRA type models is the breaking of the electroweak symmetry by radiative corrections. Here one finds two conditions: one which determines $|\mu|^2$ in terms of the soft parameters and the other which determines B (which is B_0 at the electroweak scale) in terms of $\tan \beta$, where $\tan \beta$ is the ratio of the two VEVs, i.e., $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$. Using these constraints one can choose the mSUGRA parameters to be $m_0, m_{1/2}, A_0, \tan \beta$, and $\text{sign}(\mu)$. Some of the early phenomenological implications of the SUGRA models can be found in [13] and some of the later works can be found in [14, 15, 16, 17]. Soft breaking in string models determines B . In a simple heterotic string model with compactification on three tori $T_2 \times T_2 \times T_2$ with moduli consisting of S , and $T_i = T$ ($i=1,2,3$) one has on using modular invariance the result[18]

$$B = m_0 b \frac{e^{D/2}}{(T + \bar{T})^3}, \quad (6)$$

where $D = -\log(S + \bar{S} + \dots)$, and $b = b(m_0, m_{1/2}, A_0)$ is a function of the other soft parameters $m_0, m_{1/2}, A_0$. Since $e^{-D} = 2/g_{\text{string}}^2$, one determines $\tan \beta$ in terms of α_{string} and other soft parameters. The loop corrections play an important role in

the analysis[19, 20]. It should be noted that historically the first hint that the top quark may be much heavier than was then thought (i.e., in circa 1983) came from the analysis of radiative breaking in SUGRA models [21] and should be viewed as an important triumph of the high scale SUGRA models.

Hierarchical Breaking of SUSY: In string theory it often happens that some of the extra $U(1)$ factors are anomalous. It is then permissible to add FI- D terms one for each $U(1)$. To break SUSY one may add two scalars ϕ^\pm to MSSM[22], which are singlets under SM $SU(3)_C \times SU(2)_L \times U(1)_Y$, but with charges ± 1 under $U(1)_X$. Thus adding the term $W_\pm = m\phi^+\phi^-$ to the MSSM superpotential and minimizing the full potential $\mathcal{V} = m^2(|\phi^+|^2 + |\phi^-|^2) + \frac{g_X^2}{2} \left(\sum_i Q_X^i |\tilde{f}_i|^2 + |\phi^+|^2 - |\phi^-|^2 + \xi_X \right)^2$, one finds that $Q_X^i > 0$ drives the fields to $\langle \phi^+ \rangle = 0$, $\langle \phi^- \rangle^2 = \xi_X - \frac{m^2}{g_X^2}$, $\langle F_{\phi^+} \rangle = m\sqrt{\xi_X} + \dots$ which gives $\langle D \rangle = (m^2/g_X)$ and the scalar masses m_i^2 and the gaugino masses m_λ are then given by

$$m_i^2 \simeq m^2 Q_X^i, m_\lambda \sim \frac{1}{M_{\text{Pl}}^2} \langle F_{\phi^+} \phi^- + F_{\phi^-} \phi^+ \rangle \sim m \frac{\xi_X}{M_{\text{Pl}}^2}. \quad (7)$$

In heterotic string models one finds that the FI parameter ξ at one loop is[23, 24]

$$\xi_X \sim \frac{g_X^2 \text{Trace}(Q_X)}{192\pi^2} M_{\text{Pl}}^2. \quad (8)$$

In type II string compactifications ξ_a can in principle be of any size[25].

We investigate now the sparticle mass hierarchy with many $U(1)$'s each with an FI term which gives us the scalar potential

$$\mathcal{V}_D = \mathcal{V}_D^{\text{MSSM}} + \sum_a \frac{g_a^2}{2} \left(\sum_i Q_a^i |\tilde{f}_i|^2 + \alpha_a |\phi^+|^2 - \alpha_a |\phi^-|^2 + \xi_a \right)^2. \quad (9)$$

In this case the VEVs of ϕ^\pm will absorb one FI term but the remaining set will make contributions to the scalar mass² proportional to ξ_a . Since $\langle D_a \rangle \sim \xi_a$ one finds $m_i^2 \sim \sum_a g_a^2 Q_a^i \xi_a$. Further, in heterotic string models since ξ_a can be order $O(M_{\text{Pl}}^2)$ one can generate scalar masses of order $O(M_{\text{Pl}})$. Thus with this mechanism one can generate a split SUSY scenario[26]. However, vacuum energy considerations put stringent upper limits on the scalar masses[27]. To consider this possibility we begin with the scalar potential in SUGRA and strings which is given by

$$\mathcal{V} = -\kappa^{-4} e^{-G} [G^{M\bar{N}} G_M G_{\bar{N}} + 3] + \mathcal{V}_D; \quad G = -\kappa^2 \mathcal{K} - \ln(\kappa^6 W W^\dagger), \quad (10)$$

where \mathcal{K} is the Kahler potential, and $\kappa = 1/M_{Pl}$. An important constraint here is the condition for the vanishing of the vacuum energy which is given by [27]

$$|\gamma_S|^2 + \sum_I |\gamma_I|^2 + |\gamma_+|^2 + |\gamma_-|^2 + \frac{1}{3m_{3/2}^2 M_{Pl}^2} \sum_a \frac{g_a^2}{2} D_a^2 = 1, \quad (11)$$

where $|\gamma_S|^2 = -\frac{1}{3}G^{S\bar{S}}G_S G_{\bar{S}}$, $|\gamma_I|^2 = -\frac{1}{3}G^{I\bar{I}}G_I G_{\bar{I}}$, and $|\gamma_{\pm}|^2$ are defined in a similar fashion. The above gives $\langle D_a \rangle < m_{3/2} M_{Pl}$ which implies that the scalar mass² \tilde{m}_i^2 is bounded from above so that [27]

$$\tilde{m}_i^2 \leq m_{3/2} M_{Pl}. \quad (12)$$

Eq.(12) implies that with $m_{3/2} = O(\text{TeV})$, the sfermion mass cannot exceed the value (10^{10-13} GeV). Thus unless $m_{3/2}$ itself is of size the Planck scale, one cannot get the scalar mass to be of the Planck size. For an alternate scenario for generating a mass hierarchy in soft breaking see [28].

Ellipsoidal and Hyperbolic branch of radiative breaking of the electroweak symmetry: Sugra models resolve the problem of why the Higgs mass² at low scales is tachyonic. This is done via radiative electroweak symmetry breaking (REWSB) (For a review see [29]). However, there are two branches to REWSB which are as follows: (i) Ellipsoidal Branch: When the loop correction to the effective potential are small REWSB occurs so that one has the constraint

$$\frac{m_{1/2}'^2}{a^2} + \frac{m_0^2}{b^2} + \frac{A_0^2}{c^2} \simeq 1; \quad m_{1/2}' = m_{1/2} + cA_0. \quad (13)$$

Here one finds that the soft parameters lie on the surface of an ellipsoid for a fixed μ which fixes the radii a, b, c ; (ii) Hyperbolic Branch: For large loop correction one of the terms on the right hand side of Eq.(13) can turn negative and one has a hyperbolic branch (HB) of REWSB so that [30]

$$\frac{m_{1/2}'^2}{\alpha^2(Q_0)} - \frac{m_0^2}{\beta^2(Q_0)} \simeq \pm 1. \quad (14)$$

On HB multi-TeV scalars can exist even with small fine tuning which is parametrized by μ [30]. Here one finds that m_0 can get rather large for fixed μ , which leads to multi TeV scalars. This region of multi TeV scalars is also sometimes labeled as the Focus Point region (FP). In the mSUGRA case one is dealing with the soft parameters which are universal at the high scale which we take to be the grand

unification scale. However, the nature of physics at high scales is still largely unknown. So one must also consider sugra models with non-universalities in soft breaking (NUSUGRA)[17]. Such non-universalities can arise in the Higgs sector (NUH), in the gaugino sector (NUG), and in the third generation sector (NU3). Additionally one may also consider non-universalities in the first two generations.

Implications of Brookhaven experiment on $g_\mu - 2$: SUGRA models predict the existence of a sizable correction to the muon magnetic moment on $a_\mu = (g_\mu - 2)/2$. Defining Δa_μ to be $\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM}$, one finds that the most recent analyses using the Brookhaven data gives for Δa_μ the result[31]

$$\Delta a_\mu = 27.5(8.4) \times 10^{-10}, \quad 3.3\sigma \text{ discrepancy} \quad (15)$$

Interestingly it was already predicted in the early eighties[32] within the framework of SUGRA models that the supersymmetric electroweak corrections to Δa_μ could be as large or larger than the SM electroweak corrections (For further analyses see [33]). Additionally it is known that an extension of the standard model such as the 5D models with one compact extra dimension on a half circle (S^1/Z_2) do not generate a significant contribution to Δa_μ [34]. However, some models based on extra dimensions, such as the universal extra dimension model (UED), do also allow for a sizable correction to $g_\mu - 2$ [35].

Missing link are Sparticles: If the BNL experiment holds up, i.e., a 3.3σ discrepancy is present, then within SUSY/SUGRA it is predicted that some of the sparticles have an upper bound and must be seen at the LHC. There are 32 sparticle masses in MSSM. Including certain sum rule constraints one has in excess of 10^{25} mass hierarchies. Only one of these would be realized at the LHC if the mSUGRA or some variant of it is the correct model. We focus on the first four sparticle mass hierarchies aside from the light higgs. A mapping of the parameter space of mSUGRA under constraints from experiment reduces more than 10^4 4 particle hierarchies to very few [36, 37, 38, 39] minimal sugra patterns ($mSPs$). Specifically, one finds that only sixteen 4-particle patterns survive with $\mu > 0$ (these are labeled mSP1-mSP16) and only 6 additional 4-particle patterns survive for $\mu < 0$ (mSP17-mSP22). These patterns are displayed in Table 1. Comparing with the Snowmass[40] and the Postwamp3 benchmarks[41] one finds that these cover only 5 out of the 22 patterns listed above. Thus the analysis of [36] gives a more comprehensive mapping of the parameter space of the mSUGRA model

than those given by the Snowmass benchmarks[40], Postwamp3 benchmarks[41] and the low mass (LM)/high mass (HM) benchmarks given by the CMS Collaboration. A similar mapping of NUSUGRA finds 15 additional NUSUGRA patterns which are labeled NUSP1-NUSP15 in [38]. These patterns are displayed in Table 2 and show some significant new features such as \tilde{g} being the NLSP. Analyses similar to the above can be carried out using the soft breaking in strings and in D brane models[42] and partial results were reported in [37]. In these analyses we have not taken into account the effect of CP phases on the sparticle spectrum. Such phases can affect the spectrum strongly in some cases. However, one must also impose the constraints arising from the electric dipole moments of the electron and of the neutron when the CP phases are included (For a recent review see [43]).

From models to LHC signatures: At the LHC one collides two beams of protons with a center of mass energy of 14 TeV. No matter what the model the end result will be a bunch of leptons, jets, photons and missing energy. Out of these we have to extrapolate back to determine the underlying model. For SUGRA models with R parity, the signatures necessarily include a significant amount of missing energy. In the analysis we impose the constraints from $g_\mu - 2$ experiment, $b \rightarrow s + \gamma$ [44], WMAP, and the experimental constraints from LEP and the Tevatron. We discuss some prominent signatures below. (Some recent analyses of signatures can be found in [45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58]). The first of these is the excess of trileptonic events[59] at colliders. For example, in pp collisions one has $pp \rightarrow W^{\pm*} \rightarrow \chi^\pm \chi_2^0 \rightarrow (W^\pm + \chi_1^0) + \chi_2^0$, $W^\pm \rightarrow l_1^\pm \nu, \dots$, $\chi_2^0 \rightarrow l_2^+ l_2^- \chi_1^0, \dots$. Thus the off shell production of W^* will lead to a trileptonic signature

$$p + p \rightarrow l_1^\pm l_2^\pm l_2^\mp + \text{jets} + \text{missing energy} \quad (16)$$

Additionally there are many other sources of trileptonic signals with in SUGRA models. Like sign dileptons are produced in greater abundance than in SM.

$$\begin{aligned} \tilde{g} \rightarrow \bar{c} + \tilde{c}_L \rightarrow \bar{c} + e^+ + d, \quad \tilde{g} \rightarrow c + \tilde{c}_L^* \rightarrow c + e^- + \bar{d}, \\ \tilde{g}\tilde{g} \rightarrow e^+ e^+, e^- e^-, \dots \end{aligned} \quad (17)$$

Thus one finds a significant excess of like sign dileptons relative to what one might expect in the standard model. There are actually a large number of signatures that one would generate from the LHC data. In Table 3 a list of the signatures most likely to lead to the discovery of new physics are listed.

Fuzzy signature vectors: Given a model one can define a signature vector $\xi = (\xi_1, \dots, \xi_{41})$, $\xi_i = n_i/N$, $N = \sum_i n_i$, where n_i is the number of events for i -th signature. For a pattern X one can define a fuzzy vector pattern vector

$$\Delta\xi^X = (\Delta\xi_1^X, \dots, \Delta\xi_{41}^X). \quad (18)$$

where $\Delta\xi_i^X$ is the range for signature i within the pattern X . Two patterns X and Y are distinguishable if at least one element $\Delta\xi_i^X$ does not overlap $\Delta\xi_i^Y$. Thus define

$$(\Delta\xi^X | \Delta\xi^Y) = 0(1) : \text{overlap (no overlap)}. \quad (19)$$

According to this simple criterion some patterns are distinguishable from others. The patterns are also constrained by $B_S \rightarrow \mu^+\mu^-$ data and by the dark matter cross sections[37]. We discuss the allowed parameter space consistent with dark matter constraints in further detail below.

Decoding the origin of dark matter using LHC data[60]: As is well known dark matter constitutes a significant part of our universe (for a review see[61]). For neutralino dark matter relic density constraints can be satisfied in a variety of ways. These include the coannihilation region[62, 63], the HB region[30], and the pole region[62, 64, 65]. The pole region could include the Z pole, the light Higgs pole h and the heavy CP even and the CP odd Higgs poles. One is interested in $\Omega_\chi \equiv \rho_\chi/\rho_c$ where ρ is the mass density of relic neutralinos in the universe and ρ_c is the critical mass density needed to close the universe, i.e. $\rho_c = \frac{3H_0^2}{8\pi G_N}$. Here H_0 is the Hubble parameter at current time and G_N is the newtonian constant and $\rho_c = 1.9h_0^2 \times 10^{-29}\text{gm/cm}^3$. In the analysis of $\Omega_\chi h_0^2$ we need to solve the Boltzman equation for n , the number density of neutralinos in the early universe, which is given by $\frac{dn}{dt} = -3Hn - \langle\sigma v\rangle(n^2 - n_0^2)$. In the above, n_0 is the value of n at thermal equilibrium, $\langle\sigma v\rangle$ is the thermal average of the neutralino annihilation cross section $\sigma(\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow X)$ and v is the relative $\tilde{\chi}_1^0$ velocity, and H is the Hubble parameter at time t .

The stau co-annihilation region is one of the important regions for the satisfaction of the relic density. It involves processes such as $\tilde{\tau}\tilde{\tau} \rightarrow \tau\tau$, $\tilde{\tau}\chi \rightarrow \tau Z, \tau h, \tau\gamma$, $\tilde{\tau}\tilde{\ell}_i (i \neq \tau) \rightarrow \tau\ell_i$, $\tilde{\tau}\tilde{\tau}^* \rightarrow f_i\bar{f}_i, W^+W^-, ZZ, \gamma Z, \gamma\gamma$. Here one must consider the total density $n = \sum_i n_i$ where i runs over all the sparticles that enter in the co-annihilations, where n now obeys the equation $\frac{dn}{dt} = -3Hn - \langle\sigma_{\text{eff}}v_{\text{rel}}\rangle(n^2 - n_{\text{eq}}^2)$,

$\sigma_{\text{eff}} = \sum_{ij} \sigma_{ij} \gamma_i \gamma_j$. Here σ_{ij} is the cross section for annihilation of particles i and j , and $\gamma_i = n_{\text{eq}}^i / n_{\text{eq}}$ where n_{eq}^i refers to the number density of sparticle i at thermal equilibrium. In the coannihilation region, the neutralino is mostly a bino. On the other hand in the HB region, the neutralino can have very significant higgsino content, while in other regions of the parameter space it could have varying portions of gaugino and higgsino content. It is interesting to ask if the LHC data will allow one to differentiate among the regions of the parameter space where the dark matter originates. In a recent work this issue was analyzed in some detail[60]. It was shown that the LHC data can indeed allow one to discriminate among dark matter models. Specifically, it was shown that using various signatures one can differentiate between dark matter originating on the stau coannihilation branch vs dark matter originating in the HB region.

An out of the box possibility: a 4th generation which is mirror: Essentially all of the model building within grand unification and in strings starts with the assumptions: (i) 3 generations of quarks and leptons; (ii) the gauge group $SU(3) \times SU(2) \times U(1)$ down to the electroweak scale. A relaxation of assumption (ii) with additional $U(1)$ factors leads to interesting new predictions (see, e.g., [66, 67] and references therein). However, barring few exceptions the assumption of 3 generations is often taken without reservation, . The main reasons for the 3 generation assumption include: (i) the Z-width constraint, (ii) the CKM unitarity constraints on the CKM matrix element V_{ij} (i,j=1,2,3), (iii) the constraints on oblique parameters (S,T,U), and (iv) the gauge coupling unification constraint. The constraints on extra generations have been analyzed in a number of papers[68, 69, 70] and all of them can be overcome. Thus the Z width constraint is easily overcome by making the extra generation masses greater than $M_Z/2$. Regarding the CKM unitarity constraints, a careful analysis shows that there is a window for an extra generation consistent with the limits $|V_{14}| \leq .04$, $|V_{41}| \leq .08$, $|V_{24}| \leq .17$ (see Kribs et al in [70]), and there are also windows for an extra generation in other CKM matrix elements. Actually, the most stringent constraints arise from the so called oblique parameters (S, T, U) and specifically, from the parameter S . An extra generation will contribute an amount $\Delta S = 0.21$ which is unacceptable. However, this problem too can be overcome in specific regions of the 4th generation mass parameters. Finally, the gauge coupling unification constraints for an extra generation can also be satisfied.

Let us suppose then that there is indeed a fourth generation and further that this generation rather than being sequential is a mirror generation (For early work on mirrors in model building see [71]). In this case one needs to examine the same restrictions as discussed above. Specifically the Z-width constraint, the CKM unitarity constraint, and the gauge coupling unification constraints are satisfied as for a sequential generation. To discuss the constraints on the oblique parameters, consider an $SU(2)_L$ multiplet with up and down fermion masses M_1, M_2 . The constraint on the oblique parameter S is satisfied in much the same way as for the sequential 4th generation since the correction ΔS is invariant under the transformation[72]

$$\text{fermions } (\psi_1, \psi_2) \leftrightarrow \text{mirror fermions } (\psi_2^c, \psi_1^c); Y \leftrightarrow -Y, M_1 \leftrightarrow M_2 \quad (20)$$

Suppose then that there is a large GUT group, which unifies families and which breaks leaving a certain $U(1)_F$ subgroup unbroken under which the families and mirror families are charged and their charges do not pair up. Then one or more mirror families can remain mass less along with the sequential families down to the electroweak scale. An example of the above phenomenon is the analysis of [73] for $SO(18)$. In this analysis one finds that there are $V - A$ families with charges $Q_F = -1, Q_F = 3$ and $V + A$ families with charges $Q_F = 1, Q_F = -3, Q_F = 5$. One finds that there are 3 families with charges $Q_F = 3$, two mirror families with $Q_F = 1$ and one mirror family with $Q_F = 5$ which are light. All other families and mirror families become heavy. Thus down to the electroweak scale one finds light particles some of which are families and others mirror families. These light families and mirror families eventually gain masses at the electroweak scale.

We discuss now the implication for string model building. Allowing for a light mirror generation will modify very significantly string model building. For example, in $E_8 \times E_8$ heterotic string, one considers Calabi-Yau (CY) compactifications $M_{10} = M_4 \times K$, where K is a CY manifold. The resulting theory is a 4D $N = 1$ theory with the gauge symmetry $E_8 \times E_6$. Typically one ends up with many families and mirror families, and one needs quotient manifolds K/G where G is a discrete symmetry of K which gives $n(27) - n(27^*) = \chi(K)/2N_G$, where $\chi(K)$ is Euler Characteristic, and $N(G)$ is the number of elements of G . With one light mirror generation, we should not impose the constraint $\chi(K)/2N_G = 3$ but rather $\chi(K)/2N_G = 2$. Many additional possibilities in string model building exist if one allows for a light mirror generation. For example, Kac-Moody level 2 heterotic string constructions are interesting in that they allow for adjoint Higgs

representations to break the gauge symmetry. However, no known examples of 3 massless generations exist. For this reason not much model building has occurred for this class of models. However, this class of models could become viable if one allows for a light mirror generation and three light sequential generations since $n_f - n_{mf} = 2$

One can extend MSSM to accommodate a light mirror generation[74]. Thus in analogy to the ordinary lepton generation such as the 3rd generation of leptons

$$\psi_L \equiv \begin{pmatrix} \nu_L \\ \tau_L \end{pmatrix} \sim (1, 2, -\frac{1}{2}), \tau_L^c \sim (1, 1, 1), \nu_L^c \sim (1, 1, 0), \quad (21)$$

where the quantum numbers correspond to the $SU(3)_C$, $SU(2)_L$, $U(1)_Y$ one has a mirror generation defined by

$$\chi^c \equiv \begin{pmatrix} E_{\tau L}^c \\ N_L^c \end{pmatrix} \sim (1, 2, \frac{1}{2}), E_{\tau L} \sim (1, 1, -1), N_L \sim (1, 1, 0). \quad (22)$$

Similarly corresponding to the ordinary quarks such as the third generation quarks

$$q \equiv \begin{pmatrix} t_L \\ b_L \end{pmatrix} \sim (3, 2, \frac{1}{6}), t_L^c \sim (3^*, 1, -\frac{2}{3}), b_L^c \sim (3^*, 1, \frac{1}{3}),$$

one has mirror quarks defined by

$$Q^c \equiv \begin{pmatrix} B_L^c \\ T_L^c \end{pmatrix} \sim (3^*, 2, -\frac{1}{6}), T_L \sim (3, 1, \frac{2}{3}), B_L \sim (3^*, 1, -\frac{1}{3}).$$

In order to discuss experimental implications of such an extension one needs to construct the couplings in mirMSSM involving standard model supermultiplets and mirror supermultiplets. A partial analysis of such couplings was given in [74]. Using these couplings one finds some very distinct signatures for mirMSSM. One important result is the possibility of a very large contribution to the neutrino magnetic moment and the other consists of very distinct signatures at the LHC for mirror quarks and mirror leptons.

We discuss first the τ neutrino magnetic moment including the exchange of mirror leptons and their sparticle counterparts. In the standard model

$$\mu_{\nu_\tau} = \frac{3eg^2m_\nu}{64\pi^2M_W^2} \sim (m_{\nu_\tau}/1eV) \times 3 \times 10^{-19} \mu_B,$$

where m_ν is the neutrino mass and where μ_B is the Bohr Magneton. A similar size is expected in the supersymmetric extension from the exchange of the the

charginos and sleptons. We will focus now on τ neutrino magnetic moment. The current limits on ν_τ is[75]

$$|\mu(\nu_\tau)| \leq 1.3 \times 10^{-7} \mu_B \quad (23)$$

Thus the neutrino magnetic moments predicted in the standard model lie far beyond the reach of experiment. However, the magnetic moments in mirMSSM lie within reach. For illustration we consider a simple model where there is a mixing only between the mirror generation and the 3rd generation. In this case including the mirror particle and sparticle exchange one gets for the τ neutrino magnetic moment the result[74]

$$\begin{aligned} \Delta\mu_{\nu_\tau} &\sim \frac{g_2^2 m_e m_{\tau'} \mu_B}{48\pi^2} G_1\left(\frac{m_{\tau'}}{m_W}\right) + \dots, \\ G_1(r) &= \frac{4 - r^2}{1 - r^2} + \frac{3r^2}{(1 - r^2)^2} \ln(r^2), \end{aligned} \quad (24)$$

where τ' is the mirror lepton. The modification produces a correction numerically so that a μ_{ν_τ} as large as $O(10^{-9})$ can arise and thus within the realm of experimental observation with improved experiment. At the same time one finds that the contribution of the mirrors to the τ magnetic moment is within the current experimental limits.

Next we discuss the LHC signatures for mirrors. In the analysis we will include a right handed singlet for each of the generations so that we will have Dirac neutrinos. By inclusion of a mirror generation we have added the following new set of fermionic particles: B, T, E, N , where all fields including N are Dirac. At the same time in the bosonic sector we have added the following set of scalars [74] $\tilde{B}_1, \tilde{B}_2, \tilde{T}_1, \tilde{T}_2, \tilde{E}_1, \tilde{E}_2, \tilde{\nu}_1, \tilde{\nu}_2, \tilde{\nu}_3$. The reason for the appearance of three extra sneutrino states is as follows: Within the third generation and the fourth generation, there are two Dirac neutrinos which contain four chiral states. These would lead to 4 chiral scalars. One of these in the usual sneutrino in the third generation in MSSM while the additional three sneutrinos are new and are listed as $\tilde{\nu}_1, \tilde{\nu}_2, \tilde{\nu}_3$ above. With inclusion of a mirror generation many new signatures are now possible. Thus, e.g., if $M_N > M_E + M_W$, one will have the decay signatures[74]

$$N \rightarrow E^- W^+, E^- \rightarrow \tau^- Z \rightarrow \tau^- e^+ e^-, \tau^- \mu^+ \mu^-, \tau^+ \tau^+ \tau^-. \quad (25)$$

The Drell-Yan process can generate interesting signatures. Thus, e.g., one has processes of the type[74]

$$pp \rightarrow Z^* \rightarrow E^+ E^- \rightarrow 2\tau 4l, 4\tau 2l, 6\tau, \quad (26)$$

where $l_1, l_2 = e, \mu$. In addition to the above one will have final states with taus, leptons and jets. Many other signatures are possible such as[74]

$$pp \rightarrow \tilde{\nu}_i \tilde{\nu}_i^* \rightarrow \tilde{E}_k^+ \tilde{E}_k^- W^+ W^-, \tilde{E}_k^+ E^- W^\mp \tilde{\chi}^\pm, \quad (27)$$

followed by the decays of the $\tilde{E}^+ \tilde{E}^-$ which give $\tau s + \text{leptons} + \text{jets} + E_T^{\text{miss}}$ which may have as many as 8 leptons, where all the leptons could be τs .

The couplings of the heavy CP even and CP odd Higgs to mirrors are very different from those of an sequential 4th generation. Thus the couplings of the CP odd Higgs boson A^0 to a sequential fourth generation are given by

$$\mathcal{L}_{4\text{th}} = \frac{ig}{2M_W} (m_{4d} \bar{d}_4 \gamma_5 d_4 \tan \beta + m_{4u} \bar{u}_4 \gamma_5 u_4 \cot \beta + \dots) A^0, \quad (28)$$

while for a mirror 4th generation one has[74]

$$\mathcal{L}_{\text{mir}} = \frac{ig}{2M_W} (M_B \bar{B} \gamma_5 B \cot \beta + M_T \bar{T} \gamma_5 T \tan \beta + \dots) A^0. \quad (29)$$

These give rise to decay branching ratios as follows[74]

$$\frac{\Gamma(A^0 \rightarrow u_4 \bar{u}_4)}{\Gamma(A^0 \rightarrow d_4 \bar{d}_4)} \simeq \frac{m_{d_4}^2}{m_{u_4}^2} \tan^4 \beta, \quad \frac{\Gamma(A^0 \rightarrow T \bar{T})}{\Gamma(A^0 \rightarrow B \bar{B})} \simeq \frac{m_B^2}{m_T^2} \cot^4 \beta. \quad (30)$$

The relative difference between the decay into a sequential fourth generation and into a mirror fourth generation in this case is a factor of $\tan^8 \beta$ which is a remarkable signature that separates 4th sequential generation from a 4th mirror generation.

The branching ratios of the CP odd Higgs can also provide important signatures that differentiate a fourth generation from a mirror generation. Thus one may define the ratio of branching ratios $R_{d_4/u_4}^{H^0} = BR(H^0 \rightarrow d_4 \bar{d}_4) / BR(H^0 \rightarrow u_4 \bar{u}_4)$. Using the MSSM vertices one finds

$$R_{d_4/u_4}^{H^0} = \frac{m_{d_4}^2}{m_{u_4}^2} (\cot \alpha \tan \beta)^2 P_{d_4/u_4}^{H^0}, \quad (31)$$

where α is the Higgs mixing parameter and $P_{d_4/u_4}^{H^0}$ is a phase space factor given by $P_{d_4/u_4}^{H^0} = (1 - 4m_{d_4}^2/m_H^2)^{3/2} (1 - 4m_{u_4}^2/m_H^2)^{-3/2}$. In contrast, for the decay of the CP even heavy Higgs into the mirror quarks when $(m_{H^0} > 2m_Q, Q = B, T)$ one finds [74]

$$R_{B/T}^{H^0} = \frac{m_B^2}{m_T^2} (\tan \alpha \cot \beta)^2 P_{B/T}^{H^0}. \quad (32)$$

We note that in this case the dependence on α and β is much different relative to the case when H^0 decays into sequential fourth generation quarks.

Conclusions: The main message is that the study of the sparticle landscape and of patterns can be a useful tool in extrapolating data back to theory. The landscape with $O(10^4)$ patterns for the 4 lightest sparticles reduces down just to about 50, in SUGRA models under the WMAP3, LEP and Tevatron constraints. This is a significant progress. The analysis of lepton and jet events already allows one to separate many of these patterns. Additional discrimination arises from $B_s \rightarrow \mu\mu$ process, Higgs production cross sections, and from studies of dark matter limits. Thus SUGRA models predict a candidate for dark matter which is detectable in direct detection of dark matter experiments. A combined analysis of limits from this data along with data from LHC is very powerful in limiting theory models and may even lead us uniquely to the underlying model beyond the standard model. However, one needs to keep an open mind regarding what LHC may teach us. In this regard it is desirable that one consider out of the box possibilities. mirMSSM is one such possibility discussed in this lecture.

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mSP	Mass Pattern	$\mu > 0$	$\mu < 0$
mSP1	$\tilde{\chi}_1^0 < \tilde{\chi}_1^\pm < \tilde{\chi}_2^0 < \tilde{\chi}_3^0$	Y	Y
mSP2	$\tilde{\chi}_1^0 < \tilde{\chi}_1^\pm < \tilde{\chi}_2^0 < A/H$	Y	Y
mSP3	$\tilde{\chi}_1^0 < \tilde{\chi}_1^\pm < \tilde{\chi}_2^0 < \tilde{\tau}_1$	Y	Y
mSP4	$\tilde{\chi}_1^0 < \tilde{\chi}_1^\pm < \tilde{\chi}_2^0 < \tilde{g}$	Y	Y
mSP5	$\tilde{\chi}_1^0 < \tilde{\tau}_1 < \tilde{l}_R < \tilde{\nu}_\tau$	Y	Y
mSP6	$\tilde{\chi}_1^0 < \tilde{\tau}_1 < \tilde{\chi}_1^\pm < \tilde{\chi}_2^0$	Y	Y
mSP7	$\tilde{\chi}_1^0 < \tilde{\tau}_1 < \tilde{l}_R < \tilde{\chi}_1^\pm$	Y	Y
mSP8	$\tilde{\chi}_1^0 < \tilde{\tau}_1 < A \sim H$	Y	Y
mSP9	$\tilde{\chi}_1^0 < \tilde{\tau}_1 < \tilde{l}_R < A/H$	Y	Y
mSP10	$\tilde{\chi}_1^0 < \tilde{\tau}_1 < \tilde{t}_1 < \tilde{l}_R$	Y	
mSP11	$\tilde{\chi}_1^0 < \tilde{t}_1 < \tilde{\chi}_1^\pm < \tilde{\chi}_2^0$	Y	Y
mSP12	$\tilde{\chi}_1^0 < \tilde{t}_1 < \tilde{\tau}_1 < \tilde{\chi}_1^\pm$	Y	Y
mSP13	$\tilde{\chi}_1^0 < \tilde{t}_1 < \tilde{\tau}_1 < \tilde{l}_R$	Y	Y
mSP14	$\tilde{\chi}_1^0 < A \sim H < H^\pm$	Y	
mSP15	$\tilde{\chi}_1^0 < A \sim H < \tilde{\chi}_1^\pm$	Y	
mSP16	$\tilde{\chi}_1^0 < A \sim H < \tilde{\tau}_1$	Y	
mSP17	$\tilde{\chi}_1^0 < \tilde{\tau}_1 < \tilde{\chi}_2^0 < \tilde{\chi}_1^\pm$		Y
mSP18	$\tilde{\chi}_1^0 < \tilde{\tau}_1 < \tilde{l}_R < \tilde{t}_1$		Y
mSP19	$\tilde{\chi}_1^0 < \tilde{\tau}_1 < \tilde{t}_1 < \tilde{\chi}_1^\pm$		Y
mSP20	$\tilde{\chi}_1^0 < \tilde{t}_1 < \tilde{\chi}_2^0 < \tilde{\chi}_1^\pm$		Y
mSP21	$\tilde{\chi}_1^0 < \tilde{t}_1 < \tilde{\tau}_1 < \tilde{\chi}_2^0$		Y
mSP22	$\tilde{\chi}_1^0 < \tilde{\chi}_2^0 < \tilde{\chi}_1^\pm < \tilde{g}$		Y

Table 1: Hierarchical mass patterns in mSUGRA. Y stands for appearance of the pattern for the sub case. Taken from [38].

Signature	Description	Signature	Description
0L	0 Lepton	0T	0 τ
1L	1 Lepton	1T	1 τ
2L	2 Leptons	2T	2 τ
3L	3 Leptons	3T	3 τ
4L	4 Leptons and more	4T	4 τ and more
0L1b	0 Lepton + 1 b-jet	0T1b	0 τ + 1 b-jet
1L1b	1 Lepton + 1 b-jet	1T1b	1 τ + 1 b-jet
2L1b	2 Leptons + 1 b-jet	2T1b	2 τ + 1 b-jet
0L2b	0 Lepton + 2 b-jets	0T2b	0 τ + 2 b-jets
1L2b	1 Lepton + 2 b-jets	1T2b	1 τ + 2 b-jets
2L2b	2 Leptons + 2 b-jets	2T2b	2 τ + 2 b-jets
ep	e^+ in 1L	em	e^- in 1L
mp	μ^+ in 1L	mm	μ^- in 1L
tp	τ^+ in 1T	tm	τ^- in 1T
OS	Opposite Sign Di-Leptons	0b	0 b-jet
SS	Same Sign Di-Leptons	1b	1 b-jet
OSSF	Opposite Sign Same Flavor Di-Leptons	2b	2 b-jets
SSSF	Same Sign Same Flavor Di-Leptons	3b	3 b-jets
OST	Opposite Sign Di- τ	4b	4 b-jets and more
SST	Same Sign Di- τ	TL	1 τ plus 1 Lepton

Kinematical signatures
1. P_T^{miss}
2. Effective Mass = $P_T^{miss} + \sum_j P_T^j$
3. Invariant Mass of all jets
4. Invariant Mass of e^+e^- pair
5. Invariant Mass of $\mu^+\mu^-$ pair
6. Invariant Mass of $\tau^+\tau^-$ pair

Table 2: A list of 40 counting signatures along with the kinematical signatures analyzed in [38]. Each counting signature is accompanied in addition by at least two jets. Lepton = e, μ . Taken from [38].

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